Data Analysis for Spectral Lines

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Introduction

- Spectral line VLBI much like continuum but with more channels
- Absorption line data is processed like continuum except ignoring line channels
- Emission line (maser) give some additional complications
- Give step-by-step description for maser data with polarization emphasizing differences with continuum

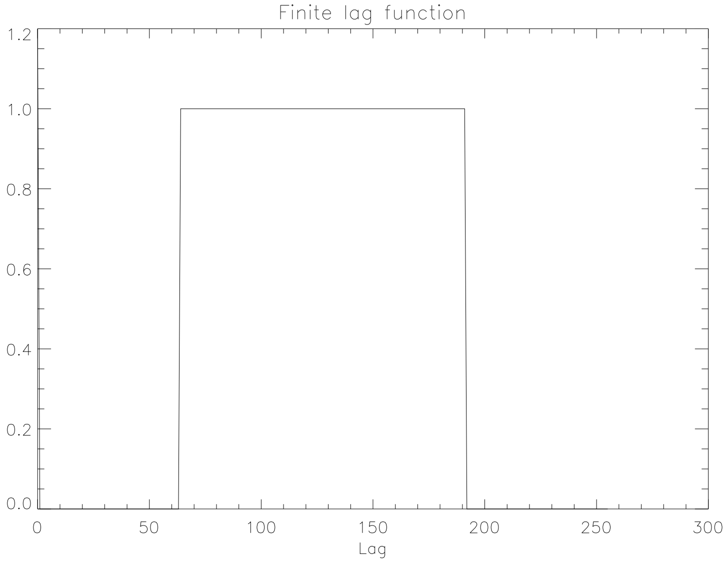
Lag – Sky Frequency Relationship

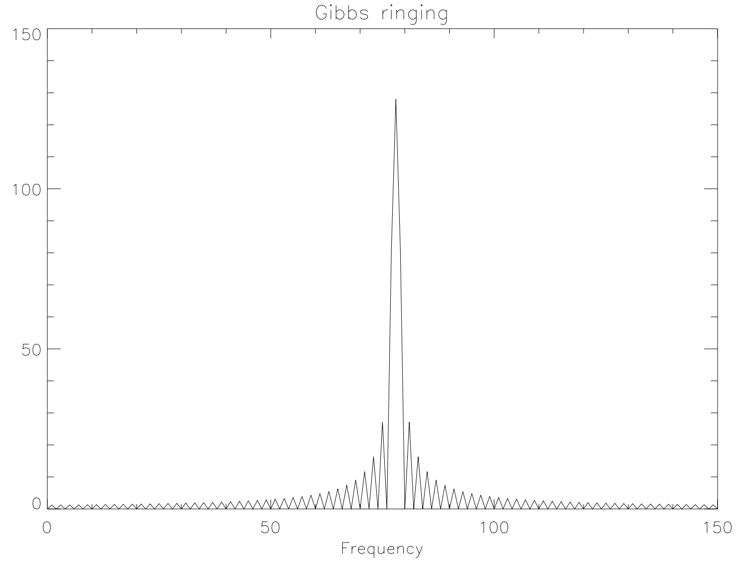
- Lag function and (Sky) frequency spectrum are related by Fourier transform
- Make spectra from lag correlator by Fourier transforming

$$V_\nu = c \int V_\tau \; e^{-2\pi\nu\tau} \; d\tau$$

Gibbs Phenomenon

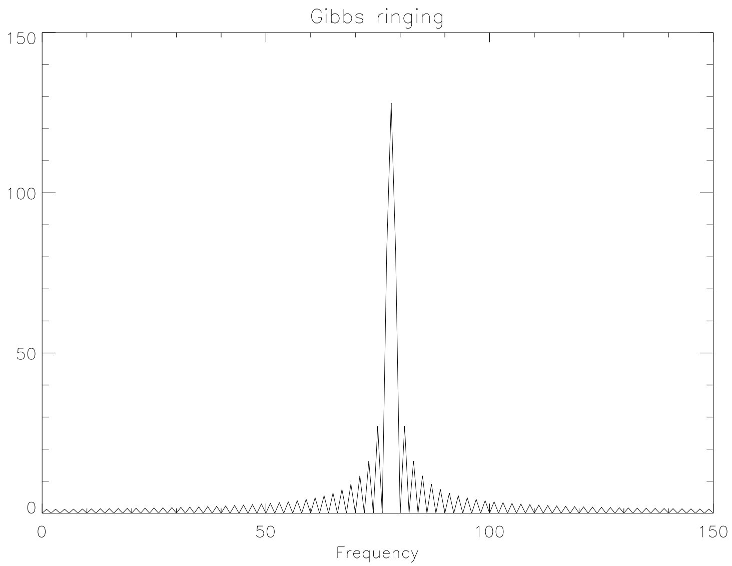
– Narrow spectral lines have very wide lag functions
– Lag function is truncated
– This leads to ringing in Fourier transform (spectrum)

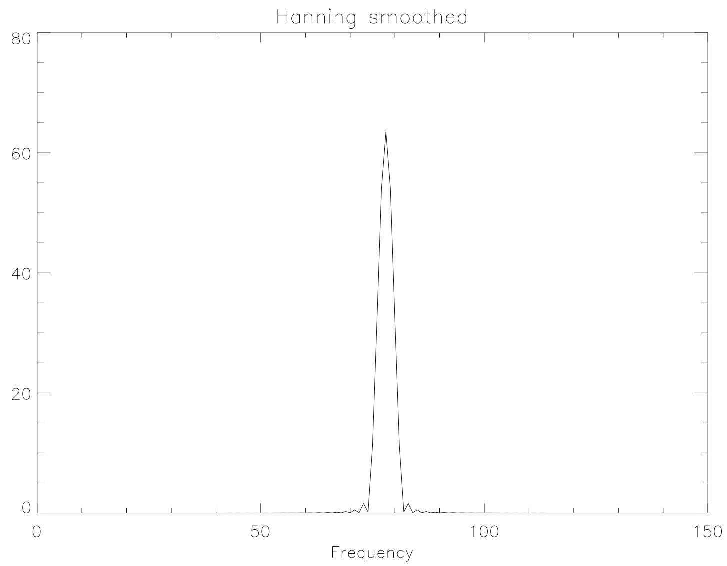




Left) Flat, finite lag function, Right) the Fourier transform

– Ringing can be reduced by Hanning smoothing but lose factor of 2 in resolution





Left) Spectrum with ringing, Right) Spectrum after Hanning smoothing

Differences with continuum:

Doppler tracking

- Doppler velocities due to Earth and Sun not small compared with resolution
- Observations usually done with fixed frequencies
- The spectra must be shifted in frequency to correct to constant velocity

Amplitude Calibration with Autocorrelations

- Autocorrelation spectra of strong masers can be used to calibrate variations in antenna gain, Tsys, etc.
- Especially useful at high frequency where amplitude calibration is difficult
- Use “template” spectrum and fit scaling to others

Phase Reference on Spectral Feature

– Maser regions contain many independent spots

– A channel with a single spot may be used at the phase reference for all channels

– Allows very accurate relative position determination

Delay Calibration

– Independent clocks, atmospheric and geometric errors cause delay errors

– Delay residuals cannot be determined from maser emission

– Must calibrate delays with a continuum source or defocus images

Step-by-Step

1) Parallactic angle correction [polarization]

– Same as continuum

This is important for polarization data and removes the phase due to apparent rotation of the feed on the sky

$$
\begin{aligned}
g_{iR} &= g_{iR}\, e^{-j\chi_i} \\
g_{iL} &= g_{iL}\, e^{j\chi_i}
\end{aligned}
$$

This causes the source response to be fixed and the instrumental poln. to rotate with parallactic angle

2) Continuum amplitude calibration

– Same as continuum
Use external calibration information to get amplitude portion of gains g
Weather corrections (opacity) needed for higher frequencies

Correlator measures ρ but want V:

$$|V| \;=\; |\rho||g_i||g_k| \text{ but for a point source } \rho \;=\; S(Jy)\,\sqrt{\frac{T^i_{sys}}{T^i_{ant}}}\sqrt{\frac{T^k_{sys}}{T^k_{ant}}}$$

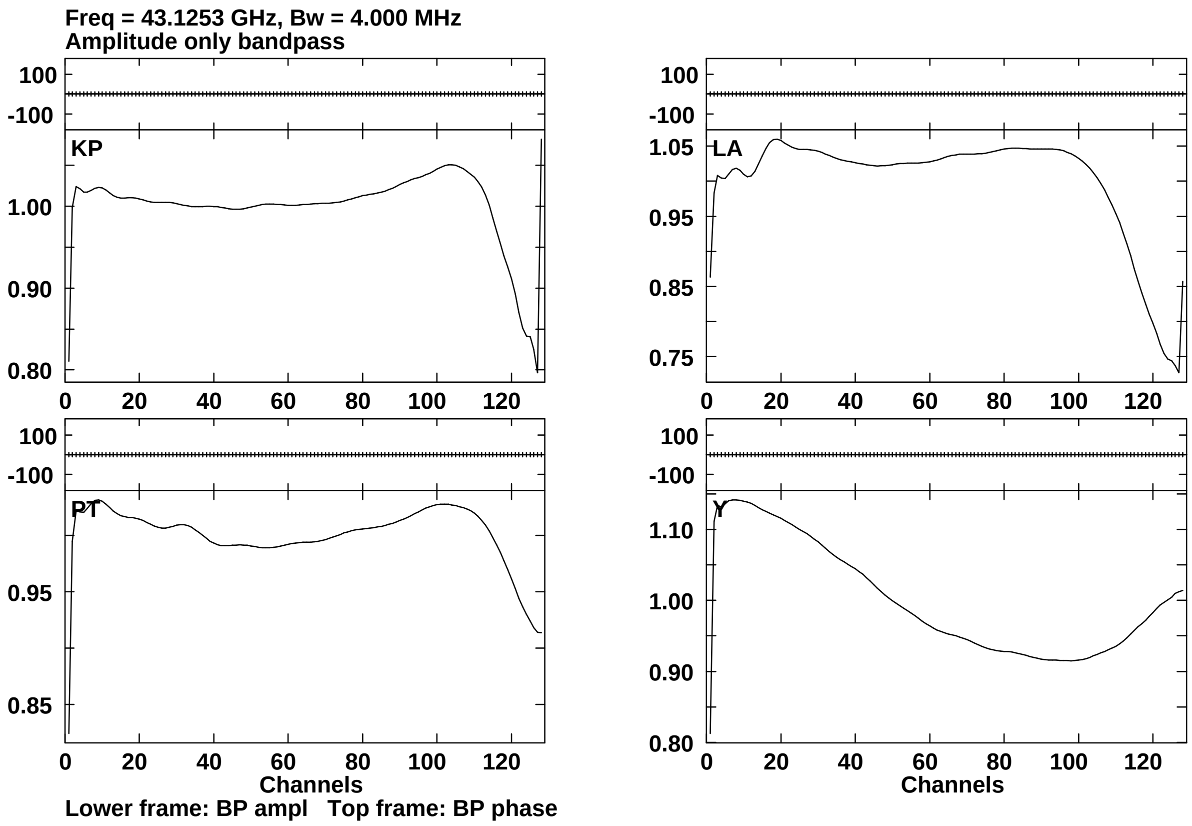
$$T_{ant} \;=\; ant\_gain(K/Jy) * S(Jy)$$

$$|g_i| \;=\; \sqrt{\frac{S(Jy)T^i_{sys}}{ant\_gain^i(K/Jy) * S(Jy)}} \;=\; \sqrt{\frac{T^i_{sys}}{ant\_gain^i(K/Jy)}}$$

3) Amplitude of Bandpass from Autocorrelation Data

– Determine amplitudes from autocorrelation data on continuum calibrator

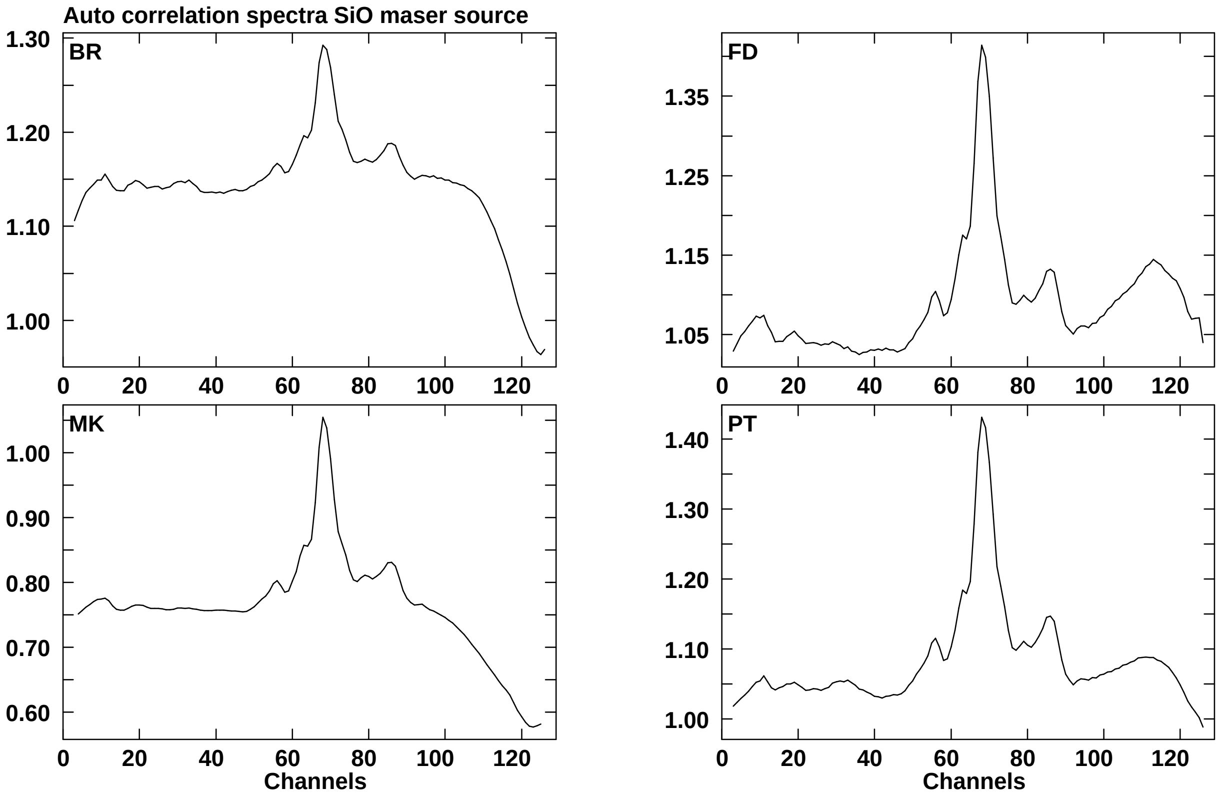
– Autocorrelations have higher SNR than crosscorrelations



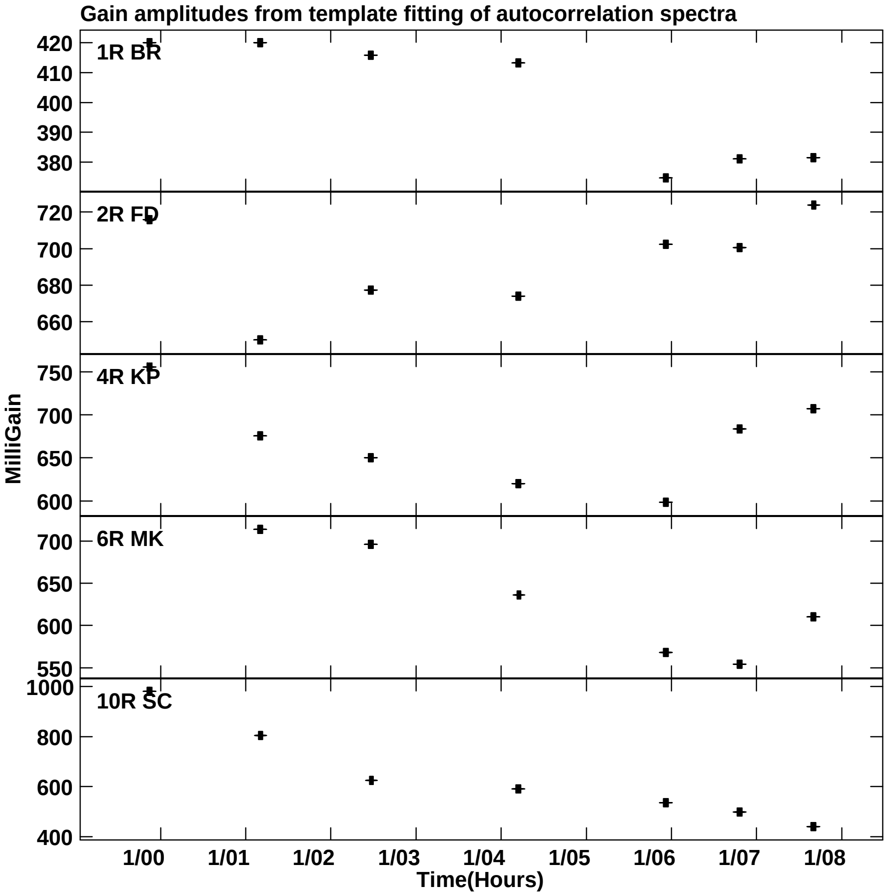
Example amplitude only bandpass functions

4) Line Amplitude Calibration from Autocorrelations

– Use spectral lines in autocorrelations to determine amplitude fluctuations
  - shift velocities to common channel
  - pick template spectra
  - determine Right/Left gain ratio of template
  - fit gain to autocorrelation spectra using template as a function of time



Example autocorrelation spectra of SiO maser source



Example gains fitted to autocorrelation spectra

5) *“Manual” phase cal*

– Same as continuum

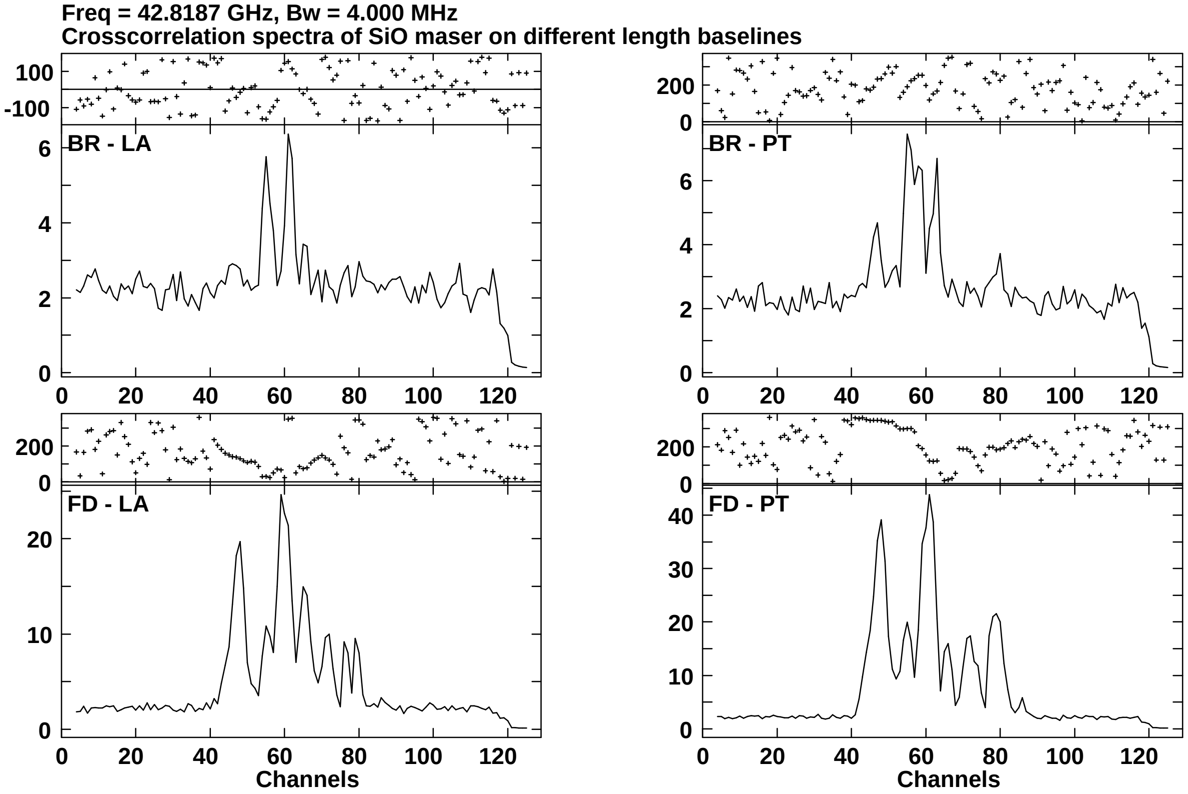
– Fringe fit a short section of good calibrator data and apply to all

– This will also refer all phase–like quantities to “reference” antenna

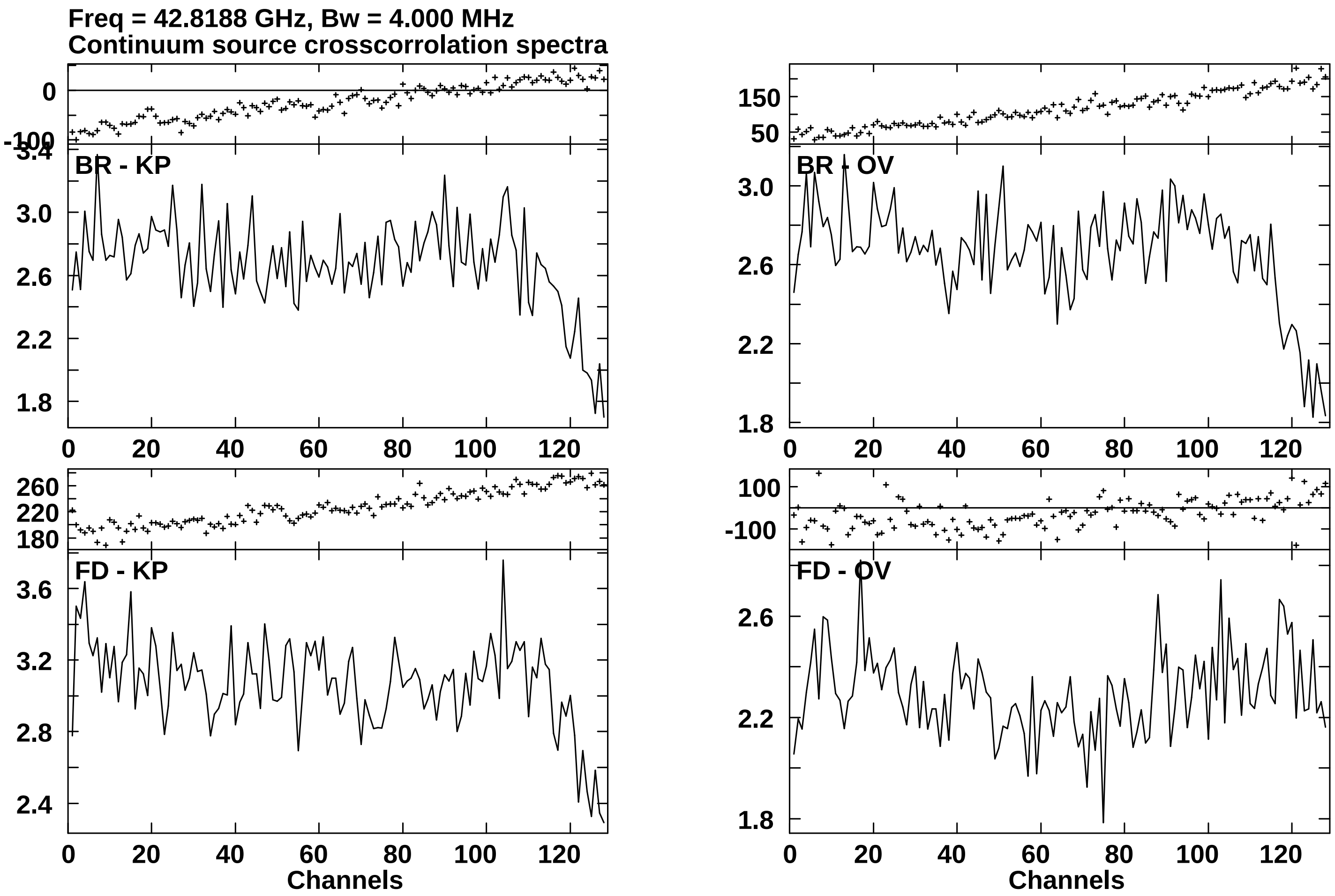
– Residual delay errors at time of calibration data will be corrected

6) Cross Correlation Continuum Fringe Fit

- Same as continuum
- Cannot Determine delay errors on line source
- Determine delay errors on continuum source to apply to line sources



Example crosscorrelation spectra of SiO maser source



Crosscorrelation spectra on continuum source showing delay residuals

7) Cross Correlation Bandpass Calibration

- (nearly) Same as continuum
- Determine bandpass phases from crosscorrelation data on a strong calibrator

8) Right-Left Delay Offset [polarization]

– Same as continuum

– There may systematic delay differences between RCP and LCP systems

– Must determine correction from cross polarized fringes on strong polarized calibrator

$$\tau_{iL} \;=\; \tau_{iL} - \tau_{R-L}$$

– Should be constant (in absence of ionospheric Faraday rotation)

– $\Delta\dot{\tau}_{R-L}$ should be 0 (in absence of ionospheric Faraday rotation)

9) Selfcal Continuum Calibrators [polarization]

– Same as continuum

– Need calibrated data on continuum sources to complete polarization calibration

10) Instrumental polarization [polarization]

– Same as continuum

– Generally need to solve for both source and instrumental polarization

- Source polarization rotates with sum of parallactic angles
- Instrumental terms rotate with difference
- Precalibration ignoring second and higher order terms:

$$
\begin{aligned}
V^{ik}_{RL} &= gain \times [V_I(d_{iR}e^{j(\chi_i-\chi_k)} + d^*_{kL}e^{-j(\chi_i-\chi_k)}) - (V_Q + jV_U)e^{-j(\chi_i+\chi_k)}] \\
V^{ik}_{LR} &= gain \times [\underbrace{V_I(d_{iL}e^{-j(\chi_i-\chi_k)} + d^*_{kR}e^{j(\chi_i-\chi_k)})}_{\text{Instrumental}} - \underbrace{(V_Q - jV_U)e^{j(\chi_i+\chi_k)}}_{\text{Source}}]
\end{aligned}
$$

– Need observations of one or more calibrators over a range of parallactic angle

– Unresolved or unpolarized source works best

11) R-L phase offset [polarization]

- Same as continuum
- Polarization angle unconstrained by total intensity calibration
- An offset in polarization angle is equivalent to right-left phase offset
- Need to know actual integrated polarization angle of a calibrator, or of a given component

$$\phi_{R-L}^{true} \;=\; 2.0 \times true\_polarization\_angle$$

- Can determine apparent R-L phase from the sums of the Q and U components from CLEAN

$$\phi_{R-L}^{apparent} \;=\; Tan^{-1}(\frac{\Sigma U}{\Sigma Q})$$

- The polarization angle calibration is completed using the gains:

$$g_{iLt} \;=\; g_{iLt}\; e^{-j(\phi_{R-L}^{true} - \phi_{R-L}^{apparent})}$$

12) Velocity shift line data and bandpass correct

– Apply bandpass calibration
– Shift line data to a consistent velocity frame
– This corrects for diurnal and annual motion of the earth

13) Apply calibration to line data

– Calibrations determined from the autocorrelations and continuum data
  - Amplitude calibration from fitting autocorrelation spectra
  - Delay calibration from continuum sources
  - Right/Left delay offset
  - Right/Left phase offset
  - Right/Left gain ratio
  - Instrumental polarization

14) Fringe fit/self cal reference channel

– Use channel with single, strong source or simple structure
– Fringe fit and self calibrate
– Apply solutions to other channels to calibrate
– If source circularly polarized (OH maser) transfer RCP solution to LCP

15) *Image*

- Image I, Q, U, V as needed for scientific goals
- Image and deconvolve each channel separately